

2022 年云南省初中学业水平考试

数学试卷

一、选择题

1. A 2. C 3. D 4. A 5. B 6. C 7. C 8. A 9. B
10. C 11. D 12. B

二、填空题

13. $x \geq -1$ 14. $(-1, 5)$ 15. $(x+3)(x-3)$

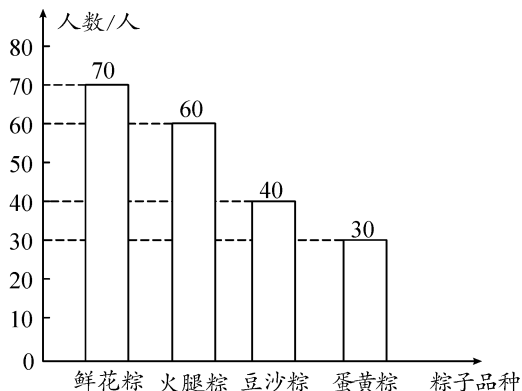
16. $x_1=1, x_2=\frac{1}{2}$ 17. 120° 18. 40° 或 100°

三、解答题

19. 解: (1) 抽样调查的总人数为 $70 \div 35\% = 200$ (人),
喜欢火腿粽的人数为 $200 - 70 - 40 - 30 = 60$ (人).
补全条形统计图如答图.

- (2) 根据题意得 $1820 \times \frac{60}{200} = 546$ (人),

答: 喜爱火腿粽的有 546 人.



第 19 题答图

20. 解: (1) 根据题意列表如下:

$a \backslash b$	1	2	3	4
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)

由表可得, (a, b) 所有可能出现的结果有 8 种.

- (2) 我认为这个游戏公平. 理由如下:

由表可得共有 8 种等可能的结果, 其中和为奇数与和为偶数的结果均有 4 种,

$$\therefore P(\text{和为奇数}) = \frac{1}{2}, P(\text{和为偶数}) = \frac{1}{2}.$$

$$\therefore \frac{1}{2} = \frac{1}{2},$$

\therefore 这个游戏公平.

21. (1) 证明: \because 四边形 $ABCD$ 是平行四边形,

$$\therefore AB \parallel CD, \therefore \angle BAE = \angle FDE.$$

$$\because E \text{ 为线段 } AD \text{ 的中点}, \therefore AE = DE.$$

$$\text{在 } \triangle BEA \text{ 和 } \triangle FED \text{ 中}, \begin{cases} \angle BAE = \angle FDE, \\ AE = DE, \\ \angle BEA = \angle FED, \end{cases}$$

$$\therefore \triangle BEA \cong \triangle FED (ASA), \therefore EF = EB.$$

$$\text{又 } \because AE = DE, \therefore \text{四边形 } ABDF \text{ 是平行四边形.}$$

$$\because \angle BDF = 90^\circ, \therefore \text{四边形 } ABDF \text{ 是矩形.}$$

- (2) 解: 由 (1) 得四边形 $ABDF$ 是矩形,

$$\therefore \angle AFD = 90^\circ, AB = DF = 3, AF = BD,$$

$$\therefore AF = \sqrt{AD^2 - DF^2} = \sqrt{5^2 - 3^2} = 4,$$

$$\therefore S_{\text{矩形 } ABDF} = DF \cdot AF = 3 \times 4 = 12, BD = AF = 4.$$

$$\because \text{四边形 } ABCD \text{ 是平行四边形}, \therefore CD = AB = 3,$$

$$\therefore S_{\triangle BCD} = \frac{1}{2} BD \cdot CD = \frac{1}{2} \times 4 \times 3 = 6,$$

$$\therefore S = S_{\text{矩形 } ABDF} + S_{\triangle BCD} = 12 + 6 = 18.$$

答: 四边形 $ABCF$ 的面积 S 为 18.

22. 解: (1) 设每桶甲消毒液的价格为 x 元, 每桶乙消毒液的价格为 y 元,

$$\text{由题意可得 } \begin{cases} 9x + 6y = 615, \\ 8x + 12y = 780, \end{cases} \text{ 解得 } \begin{cases} x = 45, \\ y = 35. \end{cases}$$

答: 每桶甲消毒液价格为 45 元, 每桶乙消毒液的价格为 35 元.

$$(2) \text{ 由题意可得 } W = 45a + 35(30 - a) = 10a + 1050,$$

$$\because 10 > 0,$$

$\therefore W$ 随 a 的增大而增大.

\because 甲消毒液的数量至少比乙消毒液的数量多 5 桶, 又不超过乙消毒液的数量的 2 倍,

$$\therefore \begin{cases} a \geq 30 - a + 5, \\ a \leq 2(30 - a), \end{cases} \text{ 解得 } 17.5 \leq a \leq 20.$$

$\because a$ 为整数, \therefore 当 $a = 18$ 时, W 取得最小值, 此时 $W = 1230, 30 - a = 12$.

答: 购买甲消毒液 18 桶, 乙消毒液 12 桶时, 才能使总费用 W 最少, 最少费用是 1230 元.

23. 解: (1) 直线 DE 与 $\odot O$ 相切. 证明如下:

$$\because BD \text{ 为 } \odot O \text{ 的直径}, \therefore \angle BCD = 90^\circ.$$

$$\because BD^2 = BC \cdot BE, \therefore \frac{BD}{BC} = \frac{BE}{BD}.$$

$$\because \angle CBD = \angle DBE, \therefore \triangle BCD \sim \triangle BDE,$$

$$\therefore \angle BDE = \angle BCD = 90^\circ.$$

$$\because \text{点 } D \text{ 在 } \odot O \text{ 上},$$

$\therefore DE$ 是 $\odot O$ 的切线, 即直线 DE 与 $\odot O$ 相切.

- (2) 成立. 证明如下:

如答图, 过点 D 作 $DG \perp PD$, 交 PC 的延长线于点 G ,

$$\therefore \angle GDP = 90^\circ.$$

$$\because \text{四边形 } ABCD \text{ 是正方形},$$

$$\therefore CD = AD, \angle ADC = 90^\circ, AC \perp BD,$$

$$\therefore \angle COD = \angle AOD = 90^\circ, \angle ADC = \angle GDP,$$

$$\therefore \angle ADC - \angle PDC = \angle GDP - \angle PDC,$$

$$\text{即 } \angle ADP = \angle GDC.$$

$$\because \angle CPD = \frac{1}{2} \angle COD = 45^\circ, \angle APD = \frac{1}{2} \angle AOD = 45^\circ,$$

$$\therefore \angle G = 90^\circ - \angle GPD = 90^\circ - 45^\circ = 45^\circ,$$

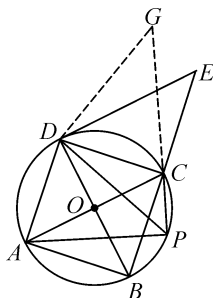
$$\therefore \angle G = \angle GPD = 45^\circ, \cos G = \frac{DG}{PG} = \frac{\sqrt{2}}{2},$$

$$\therefore DG = PD, \therefore \frac{PG}{PD} = \sqrt{2}, \therefore \frac{CG + PC}{PD} = \sqrt{2}.$$

$$\text{在 } \triangle PAD \text{ 和 } \triangle GCD \text{ 中, } \begin{cases} AD = CD, \\ \angle ADP = \angle GDC, \\ PD = DG, \end{cases}$$

$$\therefore \triangle PAD \cong \triangle GCD (\text{SAS}),$$

$$\therefore PA = CG, \therefore \frac{PA + PC}{PD} = \sqrt{2}.$$



第 23 题答图

24. 解: (1) 把点 $(0, 2)$ 代入抛物线 $y = -x^2 - \sqrt{3}x + c$ 中, 得 $c = 2$.

$$(2) \text{ 由 (1) 知, } y = -x^2 - \sqrt{3}x + 2 = -\left(x + \frac{\sqrt{3}}{2}\right)^2 + \frac{11}{4},$$

$$\therefore \text{顶点的坐标为 } \left(-\frac{\sqrt{3}}{2}, \frac{11}{4}\right).$$

\therefore 使 $S = m$ 成立的点 M 恰好有三个, 常数 $m > 0$, S 为 $\triangle ABM$ 的面积,

\therefore 其中一个点 M 就是抛物线的顶点,

$$\therefore T = -\frac{11}{4} \times 2 + \frac{11}{4} = -\frac{11}{4}.$$

$$(3) \text{ 解法一: 当 } y = 0 \text{ 时, } -x^2 - \sqrt{3}x + 2 = 0,$$

$$\text{即 } x^2 + \sqrt{3}x - 2 = 0,$$

$$\therefore x + \sqrt{3} - \frac{2}{x} = 0, \text{ 即 } x - \frac{2}{x} = -\sqrt{3}, \therefore \left(x - \frac{2}{x}\right)^2 = 3,$$

$$\therefore x^2 + \frac{4}{x^2} = 7, \therefore k^2 + \frac{4}{k^2} = 7, \therefore \left(k^2 + \frac{4}{k^2}\right)^2 = 49,$$

$$\therefore k^4 + \frac{16}{k^4} = 41,$$

$$\therefore \frac{k^4}{k^8 + k^6 + 2k^4 + 4k^2 + 16}$$

$$= \frac{1}{k^4 + k^2 + 2 + \frac{4}{k^2} + \frac{16}{k^4}}$$

$$= \frac{1}{\left(k^2 + \frac{4}{k^2}\right) + \left(k^4 + \frac{16}{k^4}\right) + 2}$$

$$= \frac{1}{7 + 41 + 2}$$

$$= \frac{1}{50}.$$

$$\text{解法二: 当 } y = 0 \text{ 时, } -x^2 - \sqrt{3}x + 2 = 0,$$

$$\text{即 } x^2 + \sqrt{3}x - 2 = 0.$$

$\therefore k$ 是抛物线 $y = -x^2 - \sqrt{3}x + 2$ 与 x 轴交点的横坐标,

即 $x = k$ 是 $x^2 + \sqrt{3}x - 2 = 0$ 的解,

$$\therefore k^2 + \sqrt{3}k - 2 = 0, \therefore k^2 = 2 - \sqrt{3}k,$$

$$\therefore k^4 = (2 - \sqrt{3}k)^2 = 4 - 4\sqrt{3}k + 3k^2 = 4 - 4\sqrt{3}k + 3(2 - \sqrt{3}k)$$

$$= 10 - 7\sqrt{3}k.$$

$$\therefore k^8 + k^6 + 2k^4 + 4k^2 + 16$$

$$= (10 - 7\sqrt{3}k)^2 + (2 - \sqrt{3}k)(10 - 7\sqrt{3}k) + 2(10 - 7\sqrt{3}k) +$$

$$4(2 - \sqrt{3}k) + 16$$

$$= 100 - 140\sqrt{3}k + 147k^2 + 20 - 24\sqrt{3}k + 21k^2 + 20 -$$

$$14\sqrt{3}k + 8 - 4\sqrt{3}k + 16$$

$$= 164 - 182\sqrt{3}k + 168(2 - \sqrt{3}k)$$

$$= 500 - 350\sqrt{3}k,$$

$$\therefore \frac{k^4}{k^8 + k^6 + 2k^4 + 4k^2 + 16}$$

$$= \frac{10 - 7\sqrt{3}k}{50(10 - 7\sqrt{3}k)}$$

$$= \frac{1}{50}.$$