

2023 年云南省初中学业水平考试

数学试卷

1. A 2. C 3. D 4. A 5. D 6. B 7. C 8. A 9. C

10. B 11. D 12. B

13.  $x \neq 10$  14. 540 15.  $(x+2)(x-2)$  16.  $\sqrt{15}$

17. 解: 原式  $= 1 + 4 - 1 + 3 - 1$

$= 6.$

18. 证明:  $\because C$  是  $BD$  的中点,  $\therefore BC = DC.$

$$\text{在 } \triangle ABC \text{ 和 } \triangle EDC \text{ 中, } \begin{cases} AB = ED, \\ AC = EC, \\ BC = DC, \end{cases}$$

$\therefore \triangle ABC \cong \triangle EDC (SSS).$

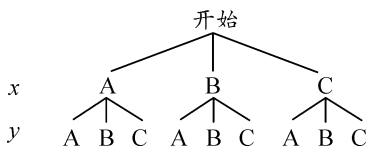
19. 解: (1)  $30 + 18 + 15 + 24 + 13 = 100$  (人).

答: 本次被抽样调查的员工人数是 100 人.

(2)  $900 \times 30.00\% = 270$  (人).

答: 估计该公司意向前往保山市腾冲市的员工人数是 270 人.

20. 解: (1) 根据题意画树状图如答图.



第 20 题答图

由树状图可知, 共有 9 种等可能的结果, 分别为  $(A, A), (A, B), (A, C), (B, A), (B, B), (B, C), (C, A), (C, B), (C, C).$

(2) 由(1)可知, 共有 9 种等可能的结果, 其中甲、乙两名同学选择种植同一种蔬菜的结果有 3 种,

$\therefore$  甲、乙两名同学选择种植同一种蔬菜的概率  $P =$

$$\frac{3}{9} = \frac{1}{3}.$$

21. 解: (1) 设每顶 A 种型号帐篷  $m$  元, 每顶 B 种型号帐篷  $n$  元,

$$\text{根据题意得 } \begin{cases} 2m + 4n = 5200, \\ 3m + n = 2800, \end{cases} \text{ 解得 } \begin{cases} m = 600, \\ n = 1000, \end{cases}$$

$\therefore$  每顶 A 种型号帐篷 600 元, 每顶 B 种型号帐篷 1 000 元.

(2) 设购买 A 种型号帐篷  $x$  顶, 总费用为  $w$  元, 则购买 B 种型号帐篷  $(20 - x)$  顶.

$\therefore$  购买 A 种型号帐篷数量不超过购买 B 种型号帐篷数量的  $\frac{1}{3},$

$$\therefore x \leq \frac{1}{3}(20 - x), \text{ 解得 } x \leq 5.$$

根据题意得  $w = 600x + 1000(20 - x) = -400x + 20000.$

$\because -400 < 0, \therefore w$  随  $x$  的增大而减小,

$\therefore$  当  $x = 5$  时,  $w$  取得最小值, 最小值为  $-400 \times 5 + 20000 = 18000$  (元),

$$\therefore 20 - x = 15.$$

答: 应购买 A 种型号帐篷 5 顶, 购买 B 种型号帐篷 15 顶, 总费用最低, 最低总费用为 18 000 元.

22. (1) 证明:  $\because$  四边形  $ABCD$  是平行四边形,

$$\therefore \angle BAD = \angle BCD, AD \parallel BC.$$

$\because AE, CF$  分别是  $\angle BAD, \angle BCD$  的平分线,

$$\therefore \angle BAE = \angle DAE = \frac{1}{2} \angle BAD, \angle BCF = \angle DCF = \frac{1}{2} \angle BCD,$$

$$\therefore \angle DAE = \angle BCF.$$

$\because AD \parallel BC,$

$$\therefore \angle DAE = \angle AEB, \therefore \angle BCF = \angle AEB,$$

$\therefore AE \parallel FC, \therefore$  四边形  $AECF$  是平行四边形.

$\because AE = AF,$

$\therefore$  平行四边形  $AECF$  是菱形.

(2) 解: 如答图, 连接  $AC.$

$\because$  四边形  $ABCD$  是平行四边形,

$$\therefore AD \parallel BC, \therefore \angle DAE = \angle AEB.$$

$\because AE$  平分  $\angle BAD,$

$$\therefore \angle BAE = \angle DAE, \therefore \angle BAE = \angle AEB,$$

$$\therefore AB = EB.$$

$$\because \angle ABC = 60^\circ,$$

$\therefore \triangle ABE$  是等边三角形,

$$\therefore \angle BAE = \angle AEB = \angle ABE = 60^\circ.$$

$\because \triangle ABE$  的面积等于  $4\sqrt{3},$

$$\therefore \frac{\sqrt{3}}{4} AB^2 = 4\sqrt{3}, \therefore AB = 4,$$

即  $AB = AE = EB = 4.$

由(1)知四边形  $AECF$  是菱形,

$$\therefore AE = CE = 4, \therefore \angle EAC = \angle ECA, BC = BE + EC = 8.$$

$\because \angle AEB$  是  $\triangle AEC$  的一个外角,

$$\therefore \angle AEB = \angle EAC + \angle ECA = 60^\circ,$$

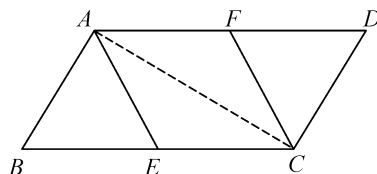
$$\therefore \angle EAC = \angle ECA = 30^\circ,$$

$$\therefore \angle BAC = \angle BAE + \angle EAC = 90^\circ,$$

即  $AC \perp AB.$

由勾股定理得  $AC = \sqrt{BC^2 - AB^2} = 4\sqrt{3},$

即平行线  $AB$  与  $DC$  间的距离是  $4\sqrt{3}.$



第 22 题答图

23. 解: (1)  $EA$  与  $\odot O$  相切, 理由如下:

如答图,连接  $OA$ .

$$\because DA \cdot AC = DC \cdot AB, \therefore \frac{DA}{DC} = \frac{AB}{CA}.$$

$\because BC$  是  $\odot O$  的直径,

$$\therefore \angle BAC = 90^\circ = \angle ADC,$$

$$\therefore \triangle ABC \sim \triangle DAC, \therefore \angle ACB = \angle DCA.$$

$$\because OA = OC,$$

$$\therefore \angle OAC = \angle ACB = \angle ACD, \therefore OA \parallel CD,$$

$$\therefore \angle OAE = \angle CDE = 90^\circ, \therefore OA \perp DE.$$

又  $\because OA$  为  $\odot O$  的半径,  $\therefore EA$  与  $\odot O$  相切.

(2) 如答图. 由 (1) 知  $OA \parallel CD$ ,

$$\therefore \triangle AOE \sim \triangle DCE, \therefore \frac{AO}{DC} = \frac{OE}{CE},$$

设  $BO = OC = OA = a$ , 则  $BC = 2a$ .

$$\because BC = BE = 2a,$$

$$\therefore S_{\triangle ABE} = S_{\triangle ABC}, EO = 3a, EC = 4a,$$

$$\therefore \frac{a}{DC} = \frac{3a}{4a}, \therefore CD = \frac{4}{3}a.$$

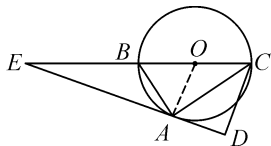
$$\because \triangle ABC \sim \triangle DAC,$$

$$\therefore \frac{BC}{AC} = \frac{AC}{DC}, \therefore AC^2 = BC \cdot DC = \frac{8}{3}a^2.$$

$$\because \triangle ABC \sim \triangle DAC,$$

$$\therefore \frac{S_{\triangle ACD}}{S_{\triangle ABC}} = \left(\frac{AC}{BC}\right)^2 = \frac{2}{3},$$

$$\therefore S_2 = \frac{2}{3}S_1, \therefore m = \frac{2}{3}.$$



第 23 题答图

24. (1) 证明: 当  $a = -\frac{1}{2}$  时, 函数表达式为  $y = 12x + 6$ ,

$$\text{令 } y = 0 \text{ 得 } x = -\frac{1}{2},$$

$\therefore$  此时函数  $y = (4a+2)x^2 + (9-6a)x - 4a+4$  (实数  $a$  为常数) 的图象与  $x$  轴有交点.

当  $a \neq -\frac{1}{2}$  时,  $y = (4a+2)x^2 + (9-6a)x - 4a+4$  为二次函数.

$$\because \Delta = (9-6a)^2 - 4(4a+2)(-4a+4) = 100a^2 - 140a + 49 = (10a-7)^2 \geq 0,$$

$\therefore$  函数  $y = (4a+2)x^2 + (9-6a)x - 4a+4$  (实数  $a$  为常数) 的图象与  $x$  轴有交点.

综上所述, 无论  $a$  取什么实数, 图象  $T$  与  $x$  轴总有公共点.

(2) 解: 存在整数  $a$ , 使图象  $T$  与  $x$  轴的公共点中有整数, 理由如下:

当  $a = -\frac{1}{2}$  时, 不符合题意;

当  $a \neq -\frac{1}{2}$  时, 在  $y = (4a+2)x^2 + (9-6a)x - 4a+4$  中,

$$\text{令 } y = 0 \text{ 得 } 0 = (4a+2)x^2 + (9-6a)x - 4a+4,$$

$$\text{解得 } x = -\frac{1}{2} \text{ 或 } x = \frac{4a-4}{2a+1}.$$

$$\because x = \frac{4a-4}{2a+1} = 2 - \frac{6}{2a+1}, a \text{ 是整数},$$

$\therefore$  当  $2a+1$  是 6 的因数时,  $\frac{4a-4}{2a+1}$  是整数,

$\therefore 2a+1 = -6$  或  $2a+1 = -3$  或  $2a+1 = -2$  或  $2a+1 = -1$  或  $2a+1 = 1$  或  $2a+1 = 2$  或  $2a+1 = 3$  或  $2a+1 = 6$ ,

$$\text{解得 } a = -\frac{7}{2} \text{ 或 } a = -2 \text{ 或 } a = -\frac{3}{2} \text{ 或 } a = -1 \text{ 或 } a = 0$$

$$\text{或 } a = \frac{1}{2} \text{ 或 } a = 1 \text{ 或 } a = \frac{5}{2}.$$

$\because a$  是整数,

$\therefore a$  的值为  $-2, -1, 0$  或  $1$ .